Biomechanical Pole Vault Model

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5/2016
Physics 499B

Abstract

A mathematical model of the takeoff phase in the pole vault for an athlete vaulting with a rigid pole is analyzed. The biomechanical model shows that there is an optimum takeoff angle of about thirty-two degrees that maximizes the vaulter’s grip height on the pole. This was calculated using the takeoff velocity, takeoff angle, the athlete’s vertical reach, and the depth of the plant box. This was also the prediction made by Nicholas Lithorne, the creator of the mathematical model. The experiment also expands on the works of Lithorne by using a more realistic moment of inertia for the calculations. The new moment of inertia shows that there is a slight drop in maximum grip height compared to the original single point mass calculation made by Lithorne.
Introduction

Pole vault is a track and field event in which an individual uses a long flexible pole as an aid to leap over a crossbar. Pole vaulting in its non-competitive form has been around for centuries. The Greeks used ‘pole vaulting’ as a way to get over enemy walls. Other civilizations also used pole vaulting to cross over natural obstacles such as rivers. In 1850, the first real event that simulated pole vaulting took place and pole vaulting has taken off since. Modern day pole vaulting is an incredibly dynamic and complex sport that requires the individual to be extremely athletic, skillful, and courageous. The pole vault can be broken down into four simple steps: the run-up, the plant, the swing, and the extension. The run-up allows the vaulter to gain speed in a controlled manner. The plant is the process of putting kinetic energy into the pole. As soon as the pole releases the potential energy, the vaulter starts to swing up as to make the most out of the energy given back by the pole. At the extension, the vaulter pulls up on the now vertical pull to get over the crossbar. As a result, the vault relies heavily on the efficiency of the transfer of energy between the vaulter and the pole. In the early days of the pole vault rigid poles were used and this caused a lot of energy dissipation. The advancements in fiberglass and carbon fiber have greatly impacted the performances of pole vaulters.

Of course, the single greatest influence on the event of pole vaulting has been the adoption of the flexible pole. Until the mid-twentieth century, vaulters used rigid poles made of steel and bamboo. While the transition of the rigid pole to the flexible pole was happening the world record improved by 45 centimeters. The record jump occurred in the span of 3 years. Currently, the average yearly improvement is only about 3 cm, so apparently the flexible pole made quite the impact. The flexible pole reduced the amount of kinetic energy lost during the
takeoff thus making it more efficient. The flexible pole also allows the vaulter to take a higher grip on the pole. Although the vaulter can now grip a lot higher, the technique of pole vaulting has not changed much over the last few centuries.

According to Lithorne, the pole vaulting technique can be described as follows, “The main aim of the run-up is to arrive at the takeoff with the maximum amount of controlled speed. The planting of the pole begins about two strides before takeoff, with the vaulter directing the tip of the pole into the takeoff box by raising the upper end of the pole with both arms simultaneously. Most vaulters use an active running takeoff with a swift extension of the takeoff leg that forcefully drives the vaulter up off the ground.” When the vaulter plants the pole, his foot is almost directly under their upper grip, and the pole strikes the back of the plant box. During this takeoff, the vaulter tries to keep their body as stiff as possible so that less energy is lost when hitting the back of the takeoff box. When the vaulter plants the pole, he creates a bending moment in the pole by keeping the lower arm in compression and the upper arm in tension. The vaulter then rotates about the shoulders and is launched up by the potential energy that was stored in the pole. The vaulter goes over the crossbar feet first as to make the most of the rotation caused by the pole’s energy.

**Theoretical Description**

The mathematical model analyzed considers an athlete vaulting with a rigid pole. The model is therefore expected to be a close approximation of the vaulters jumping with steel and bamboo poles. A schematic diagram of the takeoff phase when an athlete is vaulting with a rigid pole is shown in figure 1. The pole was modeled as perfectly rigid rod, massless. The vaulter was modeled as a point mass located midway between the vaulter’s takeoff foot and
the upper grip on the pole. The limbs of the vaulter are massless rigid fixed links. During the takeoff, the vaulter has a velocity \( v \) directed at an angle \( \phi \). The takeoff velocity refers to the velocity before the takeoff right before planting the pole. The takeoff velocity will be greater than the velocity the vaulter leaves the ground with after planting the pole. The planting of the pole causes an instantaneous change in the speed and direction of the vaulter's center of mass. The planting of the pole results in the dissipation of the velocity component parallel to \( r \) as shown in figure 1. After takeoff, the vaulter's angular velocity gradually decreases as the vaulter rises above the ground. At the top of the vault, the vaulter has a tiny amount of kinetic energy that will allow the vaulter to go over the crossbar.

Figure 1 — Schematic diagram of a pole vaulter during the takeoff phase, and at the peak of the vault, when vaulting with the maximum grip on the pole.
One would think that the vaulters velocity after takeoff would be the best measurement for vaulting performance, but it is not that readily available. The more easily accessible measurement parameter is the vaulter’s max grip height of the pole. The maximum grip height on the pole is that for which the vaulter can barely rotate to vertical with the pole. The higher the velocity after takeoff the higher the vaulter can grip on the pole. Figure 1 allows us to approximate the max grip height on the pole. If the vaulter experiences no dissipative forces, the rise in height of the vaulter’s center of mass, \( H \), from the takeoff to the peak of the pole is as follows:

\[
H = \frac{KE_{to}}{Mg} = L - h - b
\]  

(1)

Where \( KE_{to} \) is the kinetic energy of the athlete right after takeoff, \( L \) is the vaulter’s grip height, \( h \) is the vertical reach, \( b \) is the depth of the plant box, \( g \) is gravity, and \( M \) is the mass of the vaulter. The \( KE_{to} \) can be represented by equation 2:

\[
KE_{to} = \frac{1}{2} I_a \omega^2
\]  

(2)

Where \( \omega \) and \( I_a \) are the angular velocity and moment of inertia bout the takeoff box, respectively. The moment of inertia can be calculated using figure 1 and is given by equation 3:

\[
I_a = M(L^2 - \frac{3h^2}{4} - hb)
\]  

(3)

The vaulter’s angular velocity can be calculates using the model and is written as follows:

\[
Mv\left\{[L^2 - (h + b)^2]^{1/2} \sin \varphi + \left(\frac{h}{2} + b\right) \cos \varphi\right\} = I_a \omega
\]  

(4)
Combining equations 1-4, we get the result of equation 5. Equation 5 allowed me to calculate the max grip height with respect to \( h, b, v, \) and \( \phi \). This equation will allow for the calculation of the max grip height only using four variables.

\[
L - h - b = \frac{v^2}{2g} \left\{ \left[ L^2 - (h + b)^2 \right]^{1/2} \sin \varphi + \left( \frac{h}{2} + b \right) \cos \varphi \right\}^2
\]

\[\frac{M \left( L^2 - \frac{3h^2}{4} - hb \right)}{M (L^2 - \frac{3h^2}{4} - hb)} \]

The speed than an athlete can attain after takeoff decreases rapidly with increasing takeoff angle. Figure 2 represents the dissipation of velocity with respect to takeoff angle. The velocity-angle curve was used to calculate the max grip of a pole vaulter with respect to the angle. This experiment set out to prove that there is an optimum angle of \(~32\) degrees.
Figure 2 – Takeoff velocity as a function of the takeoff angle for a vaulter carrying a pole is represented by the dashed line. Using the graph I was able to calculate the velocity dissipation with a given takeoff angle.

A second part of the experiment was to make the equation for the maximum grip height more accurate. To do this, I treated the human body as four point masses, unlike Lithorne’s single point mass model. The multi-segmented model included the legs, torso, head, and the arms. Using the same approach used to find equation 5, I was able to get a new equation with a more exact moment of inertia. This equation goes as follows:

\[
L - h - b = \frac{v^2}{2g} \left\{ \left( \frac{L^2}{4} - \frac{3h^2}{4} - \frac{3hb}{2} \right) \sin \phi + \left( \frac{b}{2} + b \right) \cos \phi \right\}^{1/2}
\]

with \((M_x/M)\) being the percent mass of that given body part.

**Experimental Results/Discussion**

To prove the optimum angle I had to construct a biomechanical model of myself pole vaulting. Since I pole vault at PLU, I was able to use the facilities necessary to perform this experiment. I used the pole vault pit, a stiff pole vault pole, and a camera. I had to build a theoretical model for a person of my stature and speed. My vertical reach \((h)\) is 2.13 meters. The depth of the plant box at PLU is 0.20 meters. My approach velocity would be constrained to near 6 meters per second. Using the velocity-angle curve, I was able to calculate the percent velocity dissipation after takeoff. Inserting all the values into equation 5, I was able to plot the theoretical max grip height vs. takeoff angle. The following can be shown in figure 3.
Figure 3- Theoretical maximum grip on the pole as a function of the takeoff angle for (h=2.13m, b=0.20m). The orange line shows the max grip height of a vaulter with the approach velocity of 6m/s. The blue curve would represent the max grip height if I were able to attain a constant velocity after takeoff irrespective of the takeoff angle. This is clearly not the case, so the orange line takes into account the velocity dissipation for any given angle. As predicted, there is a sloped curve that will result in an optimum angle of ~32 degrees.

I set out to prove the optimum angle of 32 degrees by videotaping myself and analyzing the video. The trickiest part of this model was being able to have the same approach velocity on numerous jumps. If I had not been pole vaulting for eight years, this would have been nearly impossible. Having a consistent step is perhaps the most important part of pole vaulting. I got
my step down to where my velocity was about 6 meters per second. This caused some error since I was not able to perfectly achieve that velocity every time. I started my grip height at the predicted values, but soon realized it was much too high of a grip. I adjusted my grip and jumped at various angles. When I stalled at vertical, I recorded my values. Using tracker software as shown in figure 4 I was able to determine the takeoff angle.

![Image](image.jpg)

Figure 4- Using the software tracker I was able to determine the takeoff angle. I made a visible mark on my chest than used a point mass to follow its trajectory. I then tilted the axis to determine what the angle was.

Once I got all my values, I was able to insert them into equation 5. This allowed me to graph the points as shown in figure 5. The figure shows the theoretical max grip height and also the actual grip height values retrieved from the biomechanical model. As seen, I did not record any jumps above 35 degrees or below 23 degrees. This was because they were tough to replicate and they wouldn’t give me meaningful data. As seen by the yellow line there is a
pattern that seems to follow the theoretical model. The diagram appears to prove that for a rigid pole the optimum takeoff angle is about 32 degrees.

Figure 5- Theoretical maximum grip on the pole as a function of the takeoff angle for (h=2.13m, b=0.20m). The orange line shows the max grip height of a vaulter with the approach velocity of 6m/s. The yellow line shows the results from the biomechanical model for those given values.

For the second part of my research, I inserted my theoretical values into equation 6 and received the following data represented by figure 6.
Figure 6 - Maximum grip on the pole as a function of the takeoff angle for (h=2.13m, b=0.20m). The orange line shows the max grip height of a vaulter with the approach velocity of 6m/s. The black line represents the theoretical values with a more realistic moment of inertia.

The values I receive are represented by the black line on the graph. The value for the moment of inertia was greater, resulting in a lower max grip heights. Lithorne also predicted that an improved moment of inertia would cause the values to go down.

**Conclusion**

In future experiments, a model can be constructed that would use the moment of inertia for an actual human body. This could result in the theoretical model being a lot closer to the biomechanical model concerning maximum grip height. The results of this experiment
concluded that an optimum angle of ~32 degrees is necessary for maximum grip height. The work of Lithorne was expanded and improved upon. The biomechanical model allowed me to view the pole vault mathematical model in a realistic way. Calculating the optimum takeoff angle for a modern flexible pole would make for interesting research.
Work Cited